Fig. 3 Thermal conductivity for five various directions
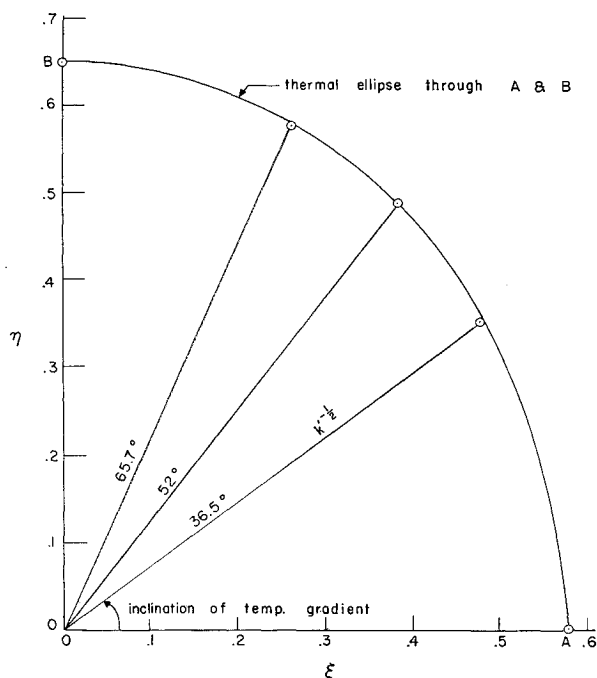

Fig. 4 Variation of thermal conductivity along the temperature gradient with direction for woven wire material 12 × 64/250 × 50

that their longer sides make angles of 30°, 45°, and 60° with the η axis. These are also the inclinations of the heat flux in the respective samples, since heat is supposed to flow parallel to the longer sides of the sample (Fig. 1). With the direction of the heat flux now known relative to the principal axes of conductivity, the direction of the temperature gradient was computed for each sample. The electrodes then were machined so that their edges were normal to the direction of the temperature gradient and then soldered to the sample. The thermal conductivity was measured at various average temperature levels, and the results are plotted in Fig. 3. The error in measurement is estimated to be ±2%.

Dividing Eq. (2) by k^1 results in

$$1 = (k_\xi/k^1) \sin^2\alpha + (k_\eta/k^1) \cos^2\alpha \qquad (5)$$

If at any point in the sample a radius vector of length $(k^1)^{-1/2}$ is measured along the direction of the temperature gradient, the locus of the tip of the radius vector relative to the principal axes of conductivity will be given by

$$1 = k_\xi \cdot \xi^2 + k_\eta \cdot \eta^2 \qquad (6)$$

Equation (6) represents an ellipse with semi-axes of lengths $k_\xi^{-1/2}$ and $k_\eta^{-1/2}$ along the ξ and η axes, respectively. This ellipse is called the thermal ellipse and is drawn in Fig. 4 with its principal axes computed from the measured values of k_ξ and k_η. The directions of the temperature gradient in the various samples also are indicated. Along each, a length is measured equal to $(k^1)^{-1/2}$, where k^1 is the respective thermal conductivity taken from the faired curves in Fig. 3 at 120°F. The experimental points deviate by less than 1% from the thermal ellipse. Thus the method of measuring thermal conductivities of metals in the form of a thin sheet by heat generation, originally expounded for thin walled tubes[3] and extended here to the case of anisotropic materials, seems to be a satisfactory as well as a convenient method.

References

[1] Carslaw, H. S. and Jaeger, J. C., *Conduction of Heat in Solids* (Oxford University Press, Oxford, England, 1948), 1st ed., p. 28.

[2] Johnson, B. V., "Heat transfer from a cylinder in crossflow with transpiration cooling," M. S. Thesis, Univ. Minnesota (1960).

[3] Allen, R. W., "Measurements of friction and local heat transfer for turbulent flow of a variable property fluid (water) in a uniformly heated tube," Ph.D. Thesis, Univ. Minnesota (1959).

Recovery of Water or Oxygen by Reduction of Lunar Rock

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The production of water or oxygen on the moon is considered from a process viewpoint. Although water would be the most desirable raw material, the actual requirement for either life support or propellant use is primarily oxygen, based on weight of material required. Since almost half the weight of the moon must be oxygen, there can be no doubt about the supply available. Several processes are considered for producing water or oxygen on the moon. The approach suggested is not dependent completely on our knowledge of moon composition, and yet it can take advantage of the optimum raw material, water, if it can be found. Processes considered vary in technological difficulty from simple rock dehydration to reduction of silicates, the objective being maximum simplicity of operation even at a cost of increased power requirement and technical problems. The criteria of weight payout time is suggested for process evaluation. This is the time required to produce a weight of product equal to the weight of plant required. A weight payout time of

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three or four months is estimated for an underground reduction process designed to minimize labor and rock handling.

Introduction

THE substantial gain in payload and extension of capabilities possible if rocket propellant could be supplied on the moon have been pointed out frequently. The advantages to any manned operation on the moon of a local supply of water or oxygen are indeed apparent. However, little work has been done in this area, because one only can speculate about what raw materials will be found on the moon.

Our knowledge of the moon will advance rapidly as initial data provide the keys necessary to predict the history of the moon. However, the manufacturing problem could continue to drag if detail prospecting is required. The purpose of this report is to suggest an approach that is not completely dependent on our knowledge of moon composition and yet can take advantage of the optimum raw material, water, if it can be found.

Water

For initial operations, the most desirable raw material would be water or ice. Recent papers offer some grounds for the hope that water or ice can be found. Kopal[1] has concluded that the interior of the moon is hot, up to 2000°K at the center. Since the initial rock probably contained water, it is assumed that large quantities of water must have been driven toward the surface or released from the moon. The fate of this water is a matter of speculation, since water has a tremendous fluxing power for silicates. Kopal suggested that some of this water may have escaped through a crack temporarily formed in the crust and could have condensed and frozen near the surface of the moon.

Salisbury[2] suggested a mechanism whereby water-rich minerals might be concentrated around the edges of crystallized lava pools, specifically beneath rilles, chain craters, and domes at maria margins. He pointed out[3] that surface domes may have resulted from the 25% increase in volume caused by the reaction, olivine + water→ serpentine + heat. Watson, Murray, and Brown[4] have shown that water released from the surface of the moon could be trapped and retained as ice at eternally shaded locations on the moon's surface.

The foregoing evidence is encouraging; however, it would seem likely that the surface ice deposits may be too small and scattered for exploitation, and that the subsurface water is mostly in the form of hydrates.

Oxygen

Although water is most desirable, the actual requirement for either life support or propellant use is primarily oxygen, based on weight of material required. Relatively light hydrogen could be brought from earth if necessary. Since almost half the weight of the moon must be oxygen, there can be no doubt about the supply available.

There can be little doubt that the subsurface lunar rock is similar to the igneous rocks of the earth's crust. Based on their dark color, the maria are considered to be made up of the heavier, basic rocks (basalt) and the "continents" to be formed from more acid magmatic rock (granite). These rocks probably lie below a vesicular crust and a porous surface layer formed by long exposure to micrometeors and radiation.

Release of oxygen from the very stable oxides and silicates that make up igneous rock is difficult and will require considerable energy, but it may be cheaper than other alternatives faced on the moon.

Processing Concepts

It is certain that the optimum moon plant will be markedly different from any earth manufacturing facility and will prob-
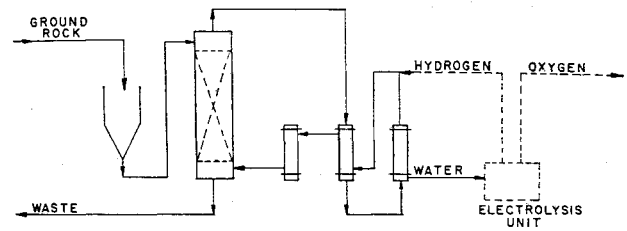

Fig. 1 Schematic diagram for the production of water by rock dehydration or production of oxygen by reduction of heavy metal oxides

ably seem contrary to our ideas of economics. The conception of air and water as precious materials requires some adjustment of our thinking. Even highly automated processes require human operations that could be very difficult on the moon.

Power, based on solar or nuclear energy, will be expensive but much cheaper than human labor. In general, a larger powerplant will be preferable to additional processing facilities.

Most of the heat generated must be radiated to space. Although the temperature may be −150°C in the shade, only a small amount of cooling can be achieved because of the low conductivity of the rock. Therefore, a high temperature process is preferable.

Mining or solid handling operations will be very difficult. Even if the operation could be completely automatic, numerous problems such as air locks and high vacuum welding or sticking of the material can be visualized.

The plant should be capable of prefabrication and testing on earth with little labor required for assembly on the moon.

In line with the foregoing concepts, several processes are considered in the following paragraphs for producing water or oxygen on the moon.

Recovery of Water of Hydration

The simplest process from the standpoint of known technology would consist of heating the rock to release chemically bound water as shown in Fig. 1. Omitting for the moment the electrolysis section shown by dashed lines, ground rock is contacted in a fluid bed column at about 1000°C with a stream of inert gas. The gas is cooled to condense out water and heated for recycle.

If a water-rich mineral could be found, as suggested by Salisbury, a yield, based on rock, of 5 to 10% water might be achieved; however, a yield of about 1% would seem more realistic for planning purposes. This low rock utilization plus extensive grinding and handling facilities required make this procedure unattractive.

Reduction of Iron Oxides

The utilization of rock could be increased from about 1%, as just described, to over 3% by using hydrogen as the circulating gas and adding an electrolysis unit as shown on Fig. 1. In this case, not only is the rock dehydrated, but also oxides of iron and possibly other heavy metals are reduced. Electrolysis of the water formed on reduction results in the net production of oxygen along with hydrogen for recycle. Water recovered from the rock should provide enough hydrogen to make up any recycle losses. Equipment might be similar to the "Nu-iron" process developed by U. S. Steel for the production of iron by hydrogen reduction. Maximum rock utilization using essentially known technology is the advantage of this process.

Electrolysis of Molten Silicates

The maximum utilization of rock and therefore minimum rock handling can be achieved through electrolysis of the molten rock to produce oxygen. It is possible that mining

could be avoided entirely through the use of a gravity flow of molten rock. Although the composition of the rock is unknown, the anode product would have to be oxygen plus any halogens that were present. Measurements to date[5] indicate that the specific conductance of molten silicates is high, the conduction being essentially ionic with good Faradaic yield of oxygen.

Metals released at the cathode would be either vaporized or dissolved in the melt. In either case, special cell design would be required to prevent the reverse reaction. Solution of metals in the melt will limit product recovery, but production of oxygen equivalent of 20% by weight of the rock would seem possible. From a long range point of view, the recovery and utilization of metals vaporized at the cathode could be considered. The power required for electrolysis is low, in the order of 12 kw-hr/lb of oxygen produced.

If large-scale production is required, electrolysis of rock offers a promising approach. It is complicated for initial small-scale production by the high viscosity and corrosiveness of the melt.

Reduction of Silicates

Utilization of rock could be increased over that obtained through hydrogen reduction of iron oxide, and uncertainties regarding composition virtually could be eliminated if silica or silicates could be reduced by hydrogen. For the reaction $\frac{1}{2} SiO_{2l} + H_{2g} \rightarrow \frac{1}{2} Si_l + H_2O_g$, the equilibrium concentration of water vapor in hydrogen is about 5 wt % water at 2500°K and 18 wt % water at 3000°K.

The practical operating temperature is limited by the volatility of the reaction product silicon, normal boiling point about 2600°K, since it is assumed that any silicon in the reaction vapor will revert to SiO_2 on cooling. The volatility of silicon will be depressed by the liquid melt and can be further offset by operating at increased hydrogen pressure.

This reduction possibly could be carried out by passing ground rock through an electric arc in a stream of hydrogen or by using a preformed electrode made of rock. A third possibility designed to avoid rock preparation entirely will be considered in the following section.

In Situ Reduction of Silicates

For initial production on the moon, it would be desirable simply to place a prefabricated unit at any random point on the surface, to connect a power line, and to have it produce oxygen without moving the rock. This idealized concept is approached in the process for underground reduction of silica as shown in Fig. 2. This process, although technically more complicated than rock dehydration, offers the possibility of minimum labor and processing facilities.

The heart of this process is the underground hydrogen arc jet, which can be similar to the constricted arc cutting torch
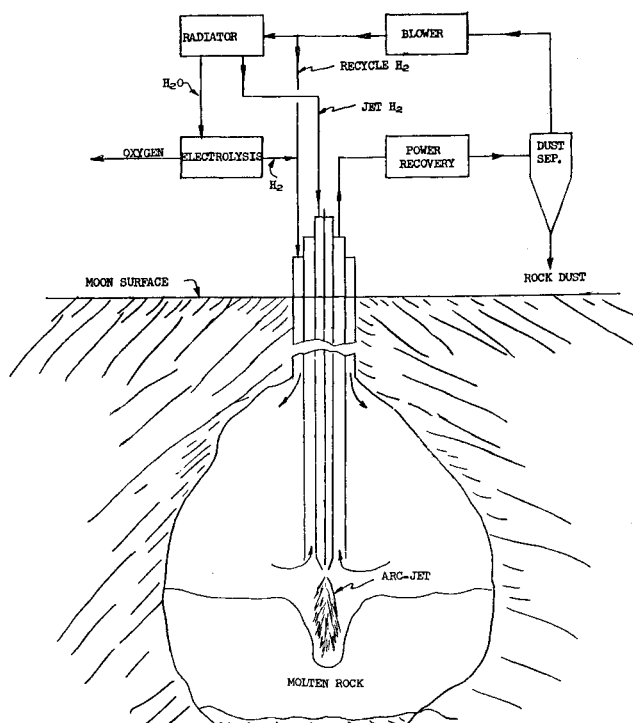
Table 1 Oxygen via in situ reduction of silica rock: process summary

Oxygen produced, lb/hr	5
Reaction temperature, °K	2500
Reaction pressure, psia	45
H_2O concentration in jet H_2, wt %	4.93
Jet hydrogen flow, lb/hr	108
Recycle hydrogen flow, lb/hr	182
Rock vaporized (as SiO_2), lb/hr	234
Arc power required, kw	750
Compressor power required, kw	30
Electrolysis power required, kw	20
Heat recovery inlet temperature, °K	1700
Heat recovery outlet temperature, °K	600
Water recovery temperature, °K	260
Weight of plant (including nuclear power source), tons	5 to 7
Payout time, weight basis, months	3 to 4

Fig. 2 Schematic diagram for the production of oxygen by hydrogen arc reduction of moon rock

or to the arc propulsion jet. To start the operation, the arc jet easily could be used to dig a hole, since the boiling point of rock at moon pressure is probably below 1000°C. After the hole is deep enough to support a few atmospheres pressure in the cavity, the top of the hole is sealed off and hydrogen circulation started through the arc, heat recovery unit, dust separator, and condensers, where water is condensed and frozen out of the hydrogen stream at −15°C. Electrolysis of the water completes the oxygen recovery, and the hydrogen formed is recycled. Assuming that rock could be melted equivalent to a sphere 20 ft in diameter and that oxygen is recovered equivalent to 5% by weight of the rock, about 44,000 lb of oxygen could be produced before moving the arc to a new hole.

As shown in Fig. 2, a secondary recycle stream of hydrogen is provided for control of the cavity temperature and rate of melting and for cooling of the jet hydrogen as required for handling in the heat recovery equipment. Assuming that the jet hydrogen leaves the molten rock at 2500°K, the use of an MHD generator for initial temperature reduction might be attractive. Otherwise, recycle hydrogen is added to bring the temperature of the jet hydrogen plus vaporized rock down to about 1700°K. The concentration of water in the hydrogen will vary as heavy metal oxides are reduced and combined water given off; however, the design concentration must be based on the final stage, reduction of silica.

Approximate process calculations were made in order to define the general area of economics of this process. The results are summarized in Table 1. It is apparent that the power requirement is high. However, it is possible that the power could be reduced considerably by operating at a slightly higher temperature and pressure. The thermal efficiency of the process will approach the power conversion efficiency, since little heat is lost except in the electric power unit.

Depending on the actual rock compositon found, some purification of the circulating hydrogen will be necessary. This offers possibilities for recovery of nitrogen and carbon dioxide, etc. Some oxygen may result from thermal decomposition of nitrites, etc. It is important, however, that the process is not greatly dependent on initial rock composition or on the many reactions that occur on heating, but de-

pends on a high temperature equilibrium that can be studied and predicted based on earth igneous rock.

In considering space manufacturing, an important criterion should be the payout time on a weight basis. This is the time required to produce a weight of product equal to the weight of plant brought from earth. The weight of plant necessary to produce 5 lb/hr of oxygen, including a nuclear power unit, is estimated to be in the order of 5 to 7 tons. Thus, such a plant would pay for itself on a weight basis in three or four months.

References

[1] Kopal, Z., "Internal structure of the moon," ARS Preprint 2455-62 (July 1962).

[2] Salisbury, J. W., "Geological problems in the location of a lunar base," *Sixth Symposium on Ballistic Missile and Aerospace Technology* (Academic Press, New York, 1961), Vol. III, pp. 311–339.

[3] Salisbury, J. W., "The origin of lunar domes," Astrophys. J. **134**, 125–129 (1961).

[4] Watson, K., Murray, B., and Brown, H., "The behavior of volatiles on the lunar surface," J. Geophys. Research **66**, 3033–3045 (1961).

[5] Bockris, J. O'M., *Modern Aspects of Electrochemistry, No. 2* (Academic Press, New York, 1959), p. 224.

A Nonlinear Lifting-Surface Theory Especially for Low-Aspect-Ratio Wings

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THIS note briefly summarizes a recently published[1] lifting-surface theory of wings with arbitrary planform in incompressible flow including nonlinear effects. Wings of small aspect ratio having sharp leading edges, as is well known, give rise to nonlinear dependence of lift and pitching moment on the angle of attack. This nonlinearity is due to the characteristic feature of the flow past a low-aspect-ratio wing, namely the strong cross flow causing the flow to separate at the leading or side edges and thus leading to the formation of free vortices on the upper surface of the wing.

All existing lifting-surface theories[2-4] are linear theories, which means that they can predict only a linear dependence of lift and pitching moment on angle of attack, because a given wing is replaced by a certain system of bound and free vortices, all of which are distributed in one plane.

The simple vortex model of the classical linear wing theory, namely one single vortex sheet, was modified in such a way that, by taking into account the special feature of the flow on low-aspect-ratio wings, a nonlinear dependence of lift and pitching moment on angle of attack could be obtained. The new vortex model used in this nonlinear theory is explained in Fig. 1. As in the linear theory of Truckenbrodt,[2] the wing is substituted by a number of elementary wings of infinitesimal span dy (Fig. 1a). Each of these elementary wings has a chordwise vortex distribution. Consequently, from each point over the chord length free vortices flow off downstream which, according to the vortex model employed, form an angle of $\alpha/2$ with the wing plane (Fig. 1b). A vortex model of this kind first was used by Bollay,[5] but for an entire single rectangular wing instead of the elementary
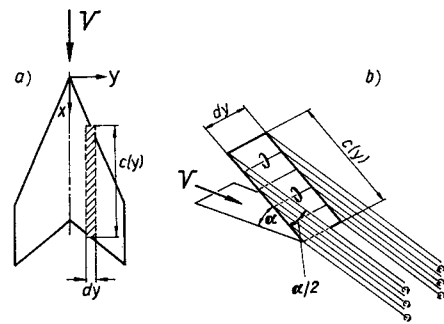
Fig. 1 Vortex model for nonlinear lifting-surface theory

wings. By this way the flowing off of free vortices at the rear at an angle of $\alpha/2$ is achieved at any point on the wing where vortex intensity is changing. In the limiting case of very small angles of attack ($\alpha \to 0$), the vortex model becomes the usual model of the linear theory.

The problem is now to determine the induced velocities at the wing area in the vortex model thus defined and to establish the flow conditions at the wing surface which then yield the equation for the unknown vortex distribution. If $k(x,y)$ is the continuous distribution of vorticity over the wing surface, the outlined vortex configuration gives the following equation for the induced angle of incidence on the wing surface:

$$\alpha_{loc} = L_1(k) + (\alpha/|\alpha|)\alpha L_2(k) + \ldots \quad (1)$$

where terms in powers of α of higher than first order are neglected. The linear operators $L_1(k)$ and $L_2(k)$ are determined by

$$L_1(k) = \frac{1}{4\pi V} \iint_S \frac{1}{y - y'} \frac{\partial}{\partial y'} \left[k(x',y') \times \left(1 - \frac{x - x'}{R}\right) \right] dx'dy' \quad (2)$$

where $R = [(x - x')^2 + (y - y')^2]^{1/2}$ and

$$L_2(k) = - \frac{1}{8V} \frac{\partial}{\partial y} \int_{x_l}^{x_t} \left(1 + \frac{x - x'}{|x - x'|}\right) \times (x - x') k(x',y)dx' \quad (3)$$

L_1 and L_2 have the following properties:

$$L(ck) = cL(k)$$
$$L(k_1 + k_2) = L(k_1) + L(k_2) \quad (4)$$

x_l, x_t are the abscissas of leading edge and trailing edge, respectively. The solution of Eq. (1) can be given in the form

$$k = k_1\alpha + k_2(\alpha/|\alpha|)\alpha^2 \quad (5)$$

Using the relations in Eq. (4), the following two equations serve for the determination of the functions k_1 and k_2:

$$L_1(k_1) = \alpha_{loc}/\alpha \quad (6)$$
$$L_1(k_2) = -(\alpha_{loc}/\alpha)L_2(k_1) \quad (7)$$

Equation (6) is identical with the integral equation of the linear theory, which can be solved by one of the well-known methods, e.g., Ref. 2. The nonlinear term is obtained from Eq. (7), which differs from Eq. (6) only by the right-hand side. Having determined k_1 from Eq. (6), it is easy to establish the right-hand side of Eq. (7), which can be solved by the same methods used in the linear theories. After functions k_1 and k_2 have been determined, the aerodynamic coefficients c_L and c_M can be obtained from the vorticity distribution $k(x,y)$ in the usual way.

By use of this method, the lift distributions and aerodynamic coefficients for series of rectangular wings, swept